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THE ASTRONOMICAL TREATISE P. RYL. 27

BY

O. NEUGEBAUER

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1. At the time when P. Ryl. 27 was published in the first volume of the Catalogue of the John Rylands Library (1911) the editors could hardly have reached a better understanding of the contents of this text than they actually did. Recently, however, Erik J. Knudtzon and the present author[1] published a new text, P. Lund 35 a, whose close relationship to P. Ryl. 27 makes it possible to give an almost complete explanation of the contents of P. Ryl. 27. When P. Lund 35 a was published the above-mentioned relationship was not realized. The present study is therefore also a supplement to the commentary on P. Lund 35.

In order to avoid constant references to the original edition of P. Ryl. 27, I shall give here a translation and commentary of the whole text, including also those parts which were already correctly understood by the editors. A few improvements in the reading of damaged numbers were possible after computation showed what to expect. These readings were checked with an excellent photograph which the John Rylands Library kindly put at my disposal.

2. I shall henceforth quote P. Ryl. 27 by R, P. Lund 35 a by L, and the Demotic P. Carlsberg 9 by C.[2]

Most numbers will be given in the sexagesimal notation. I shall separate integers from fractions by a semicolon, thus writing, e. g., 17;20 for $17\,^1/_3$. Commas will be used to separate the different sexagesimal places, e. g., 1,0,5 for 3605 or 1,10;25,30° for 70°25′30″.

[1] Knudtzon-Neugebauer, Zwei astronomische Texte, Bull. Soc. Royale des Lettres de Lund 1946—1947 II, p. 77—88.

[2] Neugebauer-Volten, Untersuchungen zur antiken Astronomie IV. Ein demotischer astronomischer Papyrus (Pap. Carlsberg 9). Quellen u. Studien z. Gesch. d. Math., Abt. B, 4 (1938), pp. 383—406.

It is often convenient to make use of a standard mathematical notation which takes account of the fact that in periodic phenomena one is often not interested in the numbers of complete periods but only in the remainder after divisions by the period. If, e. g., the sun moves a total amount of 370° or 730° in the ecliptic, it will be 10° beyond its starting point in both cases. This fact is expressed by saying that 370 and 730 are "congruent 10, modulo 360".

Finally it must be realized that the ordinary rules for addition and subtraction require for their unrestricted validity the introduction of negative numbers. Thus we write

Augustus 15 — 5 years = Augustus 10.
Augustus 3 — 5 years = Augustus — 2.

3. We shall first give a transcription of two consecutive sections of L. Restorations are not indicated because they are mathematically certain. The complete text had at least one more section of the same type, and probably several more are lost. The rules of computation being perfectly known, one could extend this scheme to any length desired.

The years in the following two sections are regnal years of Nero and Vespasian, as we shall presently prove, confirming a conjecture of the original edition. Both years and months are the years and months of the Egyptian wandering calendar, as is the standard custom in astronomical texts. The zodiacal signs begin with the vernal point, following the norm of Hipparchus and Ptolemy.

Line	Date			Longitude	
[1]	6	VII	2	♍	5;3,21,31
	7	III	5	♎	2;46,46,27
		XI	13	♏	0;30,11,23
	8	VII	16	♏	28;13,36,19
[5]	9	III	19	♐	25;57,1,15
		XI	27	♑	23;40,26,11
	10	VII	30	♒	21;23,51,7
	11	IV	3	♓	19;7,16,3
		XII	11	♈	16;50,40,59

Line	Date			Longitude	
[10]	12	VIII	14	♉	14;34,5,55
	13	IV	17	♊	12;17,30,51
		XII	25	♋	10;0,55,47
	14	X	23	♌	12;34,40,38
	1	VI	26	♍	10;18,5,34
[15]	2	II	29	♎	8;1,30,30
		XI	7	♏	5;44,55,26
	3	VII	10	♐	3;28,20,22
	4	III	13	♑	1;11,45,18
		XI	21	♑	28;55,10,14
[20]	5	VII	24	♒	26;38,35,10
	6	III	27	♓	24;22,0,6
		XII	5	♈	22;5,25,2
	7	VIII	8	♉	19;48,49,58
	8	IV	11	♊	17;32,14,54

Each of these sections will be called a "group" of 12 "steps". Looking at the dates, one immediately observes that in each group the first 11 steps amount to 248 days each, called "ordinary steps". The last step, however, which leads to a new group, is a "big step" of 303^d. Consequently each group contains a total of 3031^d.

As to the longitudes, each ordinary step amounts to 27;43, 24,56°, each big step between groups to 32;33,44,51°. Both values are taken modulo 360°. In an ordinary step, 9 complete rotations are disregarded; in a big step, 11 rotations. Using these numbers, one obtains a mean value for the increase of longitude per day of 13;10,34,52°, which shows that we are dealing with the movement of the moon.

The length of 248^d of an ordinary step is known from Babylonian astronomical texts of the Seleucid period. According to these texts the velocity of the moon returns after 248^d to the same value. Because this interval corresponds to 9 complete rotations, we obtain for one anomalistic month the length of $\frac{248}{9} = 27;33,20^d$. This value is also known from Geminus, Isag. 18 (Manitius p. 204,32 f.) as the period during which the moon returns again to smallest velocity. The derivation, however, given by Geminus as the quotient $\frac{19756}{717}$ is inaccurate because the value of this quotient is 27;33,13,18 . . . and not 27;33,20.

Similarly the interval of 303^d of a big step is also close to an anomalistic period, namely 11 anomalistic months. Using the tables in Almagest IV,4 one obtains for 303^d a movement in anomaly of —1;19,37 (mod. 360). One ordinary step, however, gives +0;6,57 and thus 11 ordinary steps give +1;16,27. This shows that the big step is inserted after 11 ordinary steps in order to compensate for the small accumulated error of the ordinary steps. For a whole group we therefore can derive the value $\frac{3031}{110}$ = 27;33,16,22, . . . for the anomalistic month. The Babylonian theory of System B is based on the value 27;33,16,27, . . . The agreement is certainly not accidental.

The preceding discussion has shown that L is based on a scheme which guaranties with a high degree of accuracy the restitution of the anomaly of the moon. The periods are chosen such that an apogee, e. g., at noontime, will fall again very close to noon. We shall demonstrate that it is exactly this scheme which is described in R. This does not, however, exhaust the contents of R. A similar scheme must have existed for latitudes. Finally, both schemes are combined with a 25-year cycle for the synodic phenomena of the moon (e. g., new moons) whose importance became evident through the publication of the Demotic text C and recently again through P. Ryl. Inv. 666.[1]

4. R. section 1 (lines 1 to 14). Translation.

1. Moon.
To the total of years add 2; divide by 25;
(multiply) the rest by 365, the cycles
of 25 by 32; then add 61;
5. the total number divide,
if possible, by 3031, and the remainder
by 248 and the remainder

[1] To be published in a forthcoming volume of the Catalogue of the John Rylands Library. I am indebted to the librarian of the John Rylands Library, the late Dr. H. Guppy, and to Prof. E. G. Turner for giving me access to this interesting text.

subtract in the case of nodes from 303, in the case of no nodes from 248 and the rest count off from the Thoth new moon and there results the day of the position according to the Egyptian (calendar). The nodes are τωϛ and(?) 14,23.

Reading of line 14. The editors write τω ξ κ(αὶ?) κδ κγ. The first three signs form one group and the last sign seems to me rather ϛ than a cursive form of ξ as the editors assume. The last number but one is certainly ιδ and not κδ.

5. Let us denote by F any phenomenon which varies periodically with the period of the lunar anomaly as period. For example, F may be the moment of lowest velocity, or the apogee of the moon, etc. We need not, however, specify F except by its period. We already know that text L concerns the dates of a phenomenon F.

Let us count time in Egyptian years of the Era Augustus. Let us furthermore assume that F had occured 61^d before Thoth 1 of the year Augustus —1. In order to be accurate, we shall consider a specific moment and choose Alexandria noon, following Ptolemy. We call this moment, Augustus —2 XI 5 noon, the "epoch" of F (cf. Fig. 1[1]). We shall demonstrate that section 1 of R tells us how to find the date of F for the year Augustus $N+1$.

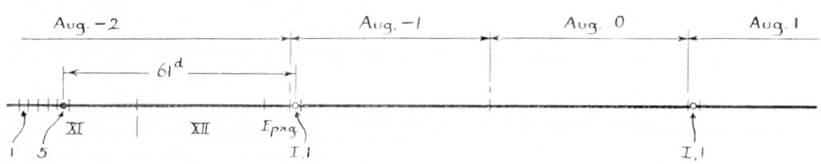

Fig. 1.

Indeed, the total of years since the starting point is obviously $n = N+2$ (line 2; cf. Fig. 2). Let α be the number of complete 25-year cycles contained in n and β the number of remaining years (β might be 0 but is certainly less than 25). Each cycle of 25 years contains 9125^d; each group contains 3031^d. Thus each

[1] Here, and in all subsequent figures, the time is not plotted in a uniform scale. Noon is indicated by a little circle, the beginning or end of day by a stroke.

cycle contains 3 groups plus 32^d. The remainder β corresponds to $\beta \cdot 365^d$ (line 3). Multiplying the quotient α by 32 (line 4) gives the excess of days over complete groups. Adding 61 days (line 4) brings us back to the epoch of F. The other end-point is the first day of the year Augustus $N+1$. Each group repeats F, normally by ordinary steps, sometimes by big steps. We have already found that n years $+ 61^d$ after the epoch correspond to a number of complete cycles, which restore F, plus

$$a = \alpha \cdot 32 + \beta \cdot 365 + 61$$

days. This number might contain γ complete groups of 3031^d (Fig. 3). Because each group restores F, we can disregard mul-

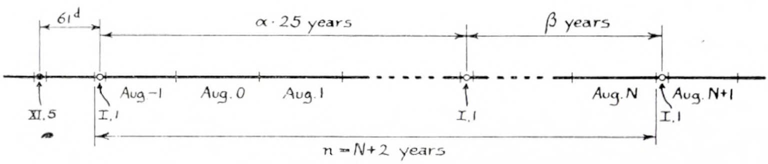

Fig. 2.

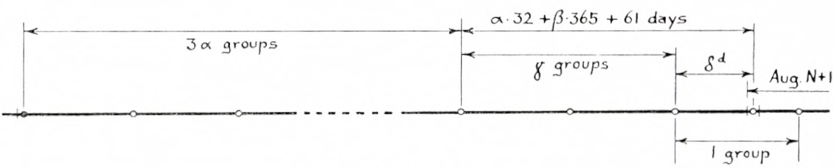

Fig. 3.

tiples of this amount. Thus we divide a by 3031 (line 6) and obtain a remainder δ between 0 (included) and 3031 (excluded). But not only whole groups restore F, but so do up to 11 ordinary steps, and finally one big step. Let us assume (Fig. 4a) that δ was found to be equal or greater than 11 ordinary steps. The text calls this "case of the nodes" (line 8). Then the first day of Augustus $N+1$ falls inside the last (big) step of our group. If we subtract from δ 11 times 248, we obtain a remainder ζ between 0 (included) and 303 (excluded). If $\zeta = 0$ we know that F falls on Thoth 1 of Augustus $N+1$. If $\zeta > 0$ we know that F falls $303 - \zeta$ days after Thoth 1 of the year $N+1$ (line 9). In the "case of no nodes" (line 9) we have found δ less than 11 ordinary steps (Fig. 4b). We divide δ by 248 (line 7) and find a quotient ε and a remainder ζ between 0 (included) and 248 (excluded). If $\zeta = 0$, the date of F is Thoth 1 of Augustus $N+1$. Otherwise $248 - \zeta$ gives the number of days between Thoth 1 and the next F (line 10).

6. Example. $N = 93$.

Thus $n + 93 + 2 = 95$. Division by 25 gives $\alpha = 3$ $\beta = 20$. Thus

$$a = 3 \cdot 32 + 20 \cdot 365 + 61 = 7457.$$

Division of a by 3031 gives $\gamma = 2$ $\delta = 1395$. Division of δ by 248 gives $\varepsilon = 5$ (thus we are in the case of "no nodes") and

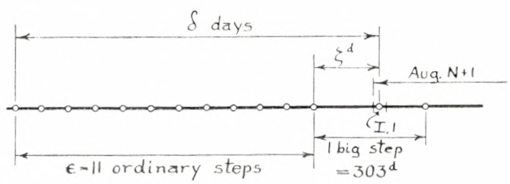

Fig. 4 a.

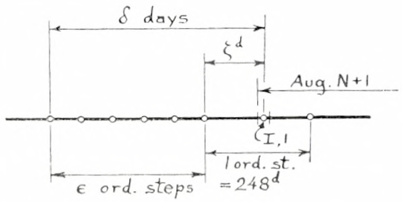

Fig. 4 b.

$\zeta = 155$. Finally we subtract 155 from 248 and obtain $93 = 3$ months + 3 days. Thus F occurs in the year Augustus $94 =$ Nero 11 on the day IV 4.

In L line [8] is found year 11 day IV 3. We could have obtained exactly the same day if we had assumed an epoch one day earlier, or, in other words, if we had counted from the last day of a year instead of from Thoth 1. The counting from Thoth 1 is not only the most natural one but is also required by all the other information to be gathered from our texts. Consequently it is tempting to ascribe the difference of one day in the dates of L to the very common mistake of substituting the "n^{th}" day for the day which is n days after the first. Yet we shall see that this discrepancy of one day plays a rôle in a later step (cf. p. 18).

Our result confirms the date Nero 11 for L line [8], proposed in the edition of L. The only other possible date would be Claudius 11, but repetition of our computation for this year shows total disagreement.

7. Second example. $N = 97$. Thus $n = 99$ $\alpha = 3$ $\beta = 24$. Hence $a = 8917$ and $\gamma = 2$ $\delta = 2855$. Because 11 ordinary steps are 2728 we have now $\varepsilon = 11$ and are dealing with the case of "nodes". Thus we must form $\zeta = 2855 - 2728 = 127$ and finally $303 - 127 = 176 = 5$ months $+ 26$ days. Consequently the date of F is Augustus $98 =$ Vespasian 1 VI 27, again in agreement with L line [14], except for the date 26 instead of 27.

Here, however, we find a disagreement between L and R in so far as our computation shows that R requires a group to end with line [13] and not with line [12] as given by L. Consequently all separating lines of L should fall one line lower according to R. In other words the values found in L just below a separating line will not be obtained from R. The value required by R would be obtained if one would add 248^{d} to the last value of a group of L instead of 303^{d}. We shall return to this fact in a later section (p. 17).

8. Line 11 implies that Thoth 1 is a new moon. From C we know that Tiberius 6 I 1 was a starting point of a 25-year cycle which restores synodic phenomena. Tiberius 6 corresponds to Augustus 49. Moving two cycles back brings us to Augustus —1, the year following the epoch in R. Because R requires for Augustus —1 I 1 a new moon, also C must concern new moons, a result found by comparison with modern calculation by the editors of C. This result is equally confirmed by P. Ryl. Inv. 666 which concerns new moons of the cycle beginning Thoth 1 of the year Augustus — 151 = Philometor 0 = Epiphanes 24.

We now can summarise the purpose of section 1. Thoth 1 of Augustus —1 = Cleopatra 21 was a new moon. A lunar phenomenon F (as we shall see presently, the apogee of the moon) fell 61 days before. We learn from section 1 how to find the date of F following Thoth 1 of a given year Augustus $N+1$.

I have omitted the last line of the section concerning "nodes", because I cannot give a satisfactory explanation of its meaning.

9. R, section 2 (lines 15 to 31). Translation.

The degrees one finds as follows. The cycles
of 25 for longitudes multiply
by 292;33,57,21°, for
latitudes by 6;38,11,24,45°.
The (number) of (intervals of) 3031 for
longitudes by 337;31,19,7°
for latitudes by 9;12,43,48,15
and the (number of intervals of) (2)48 for
longitudes by 27;43,24,56°
for latitudes by 2;43,28,34,0 and
if you subtract from 248 add
for longitudes additional 27;43,24,56°
for latitudes 2;43,28,34,0.
If you subtracted from 303 for longitudes 32;[33,4]4̣,5̣[1]
f[or longitudes . . . . .]. 1̣3;14,2̣[9,34,15]
. . . .
likewise(?) longitudes 4[9;5]7,43,50; latitudes
12;12,39,19,15; then count it off from Leo.

Readings. L. 28. μήκους $\overline{\text{λβ̣}}$ [$\overline{\text{λγ}}$]

l. 29. [$\overline{\text{μ]δ̣}}$ $\overline{\text{ν̣[α]}}$ ἐ[πὶ δὲ πλάτους . . .]. $\overline{\text{ι̣γ}}$ $\overline{\text{ιδ}}$ κ̣[θ

The editors read]ν̣γ where I would prefer]. $\overline{\text{ι̣γ}}$

l. 30. Dr. H. B. van Hoesen suggests to me αὔτ(ως) instead of αὔτ(ῶν?) of ed.

l. 30. $\overline{\text{μ̣[θ]}}$ $\overline{\text{[ν]ξ}}$ $\overline{\text{μ̣γ}}$ $\overline{\text{ν}}$

10. The second section gives rules "for longitudes" and "for latitudes" in strictly parallel form. For our discussion we separate these two problems. We begin with the longitudes.

After we have found the date of the phenomenon F for the year Augustus $N+1$ we shall now determine the corresponding longitude of the moon. The solution of this problem is based on the procedure of section 1.

From L and from line 23 of R we know that the longitude of the moon increases, modulo 360°, by

$$s = 27;43,24,56° \qquad \text{for one ordinary step}$$

and by means of L we can restore in line 28 of R the value

$$S = 32;33,44,51° \qquad \text{for one big step.}$$

Because a group consists of 11 ordinary steps and 1 big step we obtain, mod. 360°, the movement (line 20)

$$g = 337;31,19,7 \qquad \text{for one group}$$

and (line 17)

$$G = 292;33,57,21 \qquad \text{for three groups.}$$

From section 1 we know that $n = N + 2$ years contain α 25-year cycles. Because each cycle contains 3 groups we have at least 3α groups, corresponding to a movement of αG (lines 15 to 18). We have furthermore found (p. 8) the number γ of complete groups in the remaining years by dividing the corresponding number of days by the length 3031^d of a group (line 19). Thus

$$\alpha G + \gamma g$$

accounts for all complete groups. There might be, however, a remaining amount of δ days until Thoth 1 of the year $N+1$. If δ is greater than $\varepsilon = 11$ ordinary steps we have to subtract the excess ζ from 303 days (line 28) in order to reach the date of F in the year $N+1$. In this case F is the endpoint of a big step and we have not only to add εs (lines 22 and 23) but also S (line 28)[1]. If δ contains $\varepsilon < 11$ ordinary steps plus ζ days (< 248) we have similarly to add s to εs (lines 25 and 26).

11. This procedure gives the total increase in longitude during the time of the epoch until the first occurrence of F in the year Augustus $N+1$. All that we need to obtain the longitude λ for this second point is the longitude λ_0 at epoch. From line 31 we see that longitudes are counted from Leo. In this norm we have

$$\lambda_0 = \text{♌}\ 310;2,16,10 = \text{♌} - 49;57,43,50.$$

The amount of $49;57,43,50°$ has therefore to be subtracted from our total movement (line 30) in order to give λ, counted from ♌ 0°. In the ordinary norm we have

$$\lambda_0 = \text{♊}\ 10;2,16,10.$$

12. Example. $N = 93$ as in No. 6 p. 9. We found $\alpha = 3$ $\gamma = 2$. Thus

$$\alpha G = 3 \cdot 292;33,57,21 = 157;41,52,3 \text{ mod. } 360$$

[1] The total, in this case, is of course g.

$$\gamma g = 2 \cdot 337;31,19,7 = 315;2,38,14 \text{ mod. } 360.$$

Thus a total of 112;44,30,17. Because $\varepsilon = 5$ we are in the case of "subtracting from 248" and have to form

$$(\varepsilon + 1)s = 6 \cdot 27;43,24,56 = 166;20,29,36.$$

Added to the previous amount we find for the movement since epoch 279;4,59,53. From this we subtract for the epoch 49;57, 43,50 and obtain

$$\lambda = \Omega\ 229;7,16,0 = \text{♓}\ 19;7,16,3$$

exactly as in L line [8].

Thus also the longitudes in L are computed with the rules of R, of course with the same exception of the lines following separating lines in L as we have seen in the case of the dates.

13. We can now determine the character of F because we know the date of the epoch

Augustus −2 = Nabonassar 716 XI 5.

Using the tables of the Almagest[1] we obtain for Nabonassar 716 XI 5 Alexandria noon for the mean moon

Longitude	♊ 13;20
Anomaly	354;28
Argum. of lat.	185;51
Elongation	336;47

This already shows the following. The anomaly of the moon is so close to 360, which corresponds to the apogee, that it is evident that the "phenomenon F" is the smallest lunar velocity. For the longitude R uses ♊ 10;2,16,10. According to the Almagest the mean moon would be 3;18 farther ahead which would correspond exactly to the motion during 6 hours. This might suggest the use of morning epoch, corresponding to the practice in Egypt. In spite of this apparent agreement, a noon epoch is astronomically so much more convenient that I am inclined to explain the small discrepancy between the elements of R and the values obtained from the Almagest as the result of the use of slightly different tables.[2]

[1] THEON's "Handy Tables" lead to only slightly different values.

[2] It is clear from L that we are dealing only with mean motions. If one nevertheless computes the true moon, one obtains from Almagest IV,10 the longitude ♊ 13;49 and with the second anomaly ♊ 14;21 (Almagest V,8).

14. Section 2 of R contains also rules for "latitude" (πλάτος). From the fact that numbers greater than 5 occur[1] in this connection it is evident that πλάτος does not correspond to the latitude of modern terminology but that it represents a quantity analogous to our "argument of latitude". In other words, not the deviation from the ecliptic is measured but the distance from some point in the orbit of the moon. This is also the case in the Almagest where the argument of latitude, called πλάτος, is counted from the "northern limit" (βόρειον πέρας), i. e., from the point of greatest positive latitude (5°).

It is off-hand clear, however, that Ptolemy's definition cannot be identical with the meaning of "latitude" in R. From lines 30 and 31 we see that both longitudes and "latitudes" are counted from Leo. From the same passage we see that at epoch the "latitude" had the value

$$\beta_0 = ♌ - 12;12,39,19,15 = ♋\ 17;47,20,40,45$$

and we shall confirm this interpretation of line 31 independently (p. 19). Thus πλάτος must be used here in the same sense as in the "Handy Tables" where the numbers τοῦ βορείου πέρατος give the longitude of the "northern limit", counted backwards from ♈ 0°.[2] In other words, we should expect that β_0 gives the position of a characteristic point of the lunar orbit, probably of the northern limit, or perhaps of the diametrically opposite points or of the nodes.

This expectation is not fulfilled. We know from p. 13 that the moon at epoch was about 186° ahead of the northern limit, thus about at greatest negative latitude. The corresponding longitude was about ♊ 10, thus the ascending node was about ♍ 10°. The point β_0, however, lies near ♋ 18°, thus somewhere halfway between greatest negative latitude and the ascending node. I have no explanation to offer for this fact.

15. For the moment we do not need to know the astronomical meaning of β. All that we must assume is that it is a quantity which increases uniformly with time. If we are given the increase of β during an ordinary step of 248^d and during a big step of

[1] 6;38,.. in line 18, 9;12,... in line 21, 12;12,.. in line 31, 13;14,. (?) in line 29.

[2] Cf. Heiberg II p. 171.

303^d we should be able to compute β exactly in the same way as we did before in the case of λ. The strict parallelism between λ and β in the text of section 2 supports this expectation. Unfortunately we shall only meet with partial success in following this line of attack.

From lines 24 or 27 we see that the increase b of β during an ordinary step is given by

$$b = 2;43,28,34,0.$$

The value B for a big step was certainly given in line 29 but only].3,14,.[is still fairly well preserved. In line 21, however, we find the value

$$k = 9;12,43,48,15$$

for one group and in line 18 the value

$$K = 6;38,11,24,45$$

for 3 groups. Obviously one should expect $K = 3k$ but this relation only holds for the fractional part, not for the integers, where we have 6 instead of 27. A similar discrepancy will be found for all other values of β. We see this immediately when computing B from k and b. Indeed, we should have for one group

$$k = 11\,b + B$$

Using the above values for k and b we obtain for B the fractional part ;14,29,34,15 which explains the 14 in line 29. The integers, however, should end in 9 whereas we find in line 29 a 3 or perhaps 13 preceding 14.

It is evident that the explanation of this fact ought to be sought in a reduction of values of β modulo a certain integer. I did not succeed, however, in determining its value nor to find any plausible explanation for such a reduction.

16. R, section 3 (lines 32 to 41). Translation.

Another (and) shorter method from the beginning. {25}
All the years from Commodus, add 92, divide
by 2(5), (multiply) the remaining years by 365, the
cycles of 25 by 32, adding up
the whole numbers, divide, if possible, by

3031, the remainder by (2)48 and as much as the
final remainder will be, so much
will be missing from 293 and this count off
40. from Thoth 1 and there will result the date of the position
according to the Egyptian (calendar).

Reading. Line 32. {κε}. Smyly suggested the reading κε (φαλαίωσον) "sum up(?)". There is no doubt, however, that κε is marked as number. The scribe probably thought of the next operation to be carried out, namely the division by 25.

17. The text is not right when it calls this a "shorter" method. All that happens is that the era is changed from Augustus to Commodus and a shift occurs in the epoch. The date of the epoch is to be found directly from line 33. If we are dealing with a year Commodus N and if $n = N + 92$ is the number of years which must be used in our computations, it is clear that the epoch falls 92 years before Commodus 1 = Augustus 190. Hence the epoch is the day of the occurrence of F preceding Thoth 1 of Commodus —91 = Augustus 98 = Vespasian 1. This day is already known to us from L line [14] to be Nero 14 X 23. This day precedes the next Thoth 1 by 73 days.

In order to find the date of F in the year Commodus $N+1$ we follow closely the old scheme (cf. No. 5 p. 7 ff.). We divide $n = N + 92$ by 25 and find the quotient α and the remainder β. Then we form

$$a = \alpha \cdot 32 + \beta \cdot 365$$

(lines 33 to 35). This total of days must be divided by 3031 (line 37) which is the length of a group and the remainder δ by 248 (line 37) in order to find the number of ordinary steps. Now a distinction should be made as to the number ε of ordinary steps being either 11 or less than 11. This part of section 1 is not repeated here. The remainder ζ in this last division gives the number of days between the last occurrence of F and Thoth 1 of Commodus $N+1$ under the assumption that Thoth 1 Vespasian 1 coincides with an occurrence of F. Actually, however, this is not the case (cf. Fig. 5). As we have seen, the date of the epoch is Nero 14 X 23, thus 73 days before Vespasian 1 I 1. Consequently ζ measures only the distance from F to a day 73 days before Thoth 1 of Commodus $N+1$. The date of this day can be called Thoth 293 of year N. Thus the last occurrence of F in the year Commodus N has the date Thoth $293 - \zeta$, as stated in lines 39 and 40 (cf. Fig. 6).

The preceding discussion shows that the new method is in principle exactly the same as the old one. The only difference consists in a change of era and epoch. Furthermore, the clear distinction between the case of ordinary and big steps is omitted.

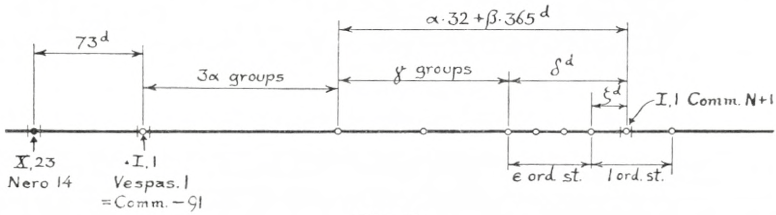

Fig. 5.

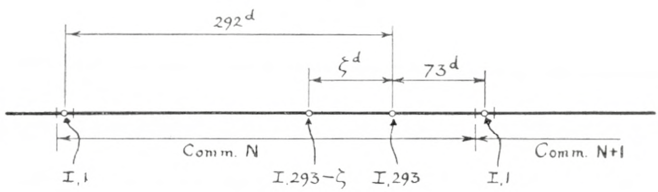

Fig. 6.

Finally, the correction for epoch is applied at the end instead of at the beginning. This, too, is no improvement because the final addition of 73 days to the remainder ζ might total more than a whole step, in which case one would not obtain the last occurrence of F but only the next to the last.

While we can say that, on the whole, section 3 is only a trivial modification of section 1, one important bit of information is concealed in this change of epoch. We have stated that the rules of section 1 lead to the values found in L, with the sole exception of the lines directly following a separating line between groups. The rules of section 3, however, are based on the epoch Nero 14 X 23 and this date is exactly a date of L after a separating line. Thus we see that somewhere between the earlier epoch Cleopatra 20 XI 5 and the new epoch a big step must purposely have been moved one line backwards.

Similarly we now find exact agreement with the dates of L though we have seen (p. 9) that the rules of section 1 would lead to dates one day higher. Thus we see not only that the separating lines between groups were moved one line down but also that all dates were lowered by one day. In other words, L was computed with the norm of section 3 and not of section 1.

It is plausible to assume that these changes were intended to correct a small accumulated error.

18. R, section 4 (lines 41 to 50). Translation.

The degrees are determined as follows. The cycles of 3(0)31 multiply for longitudes by 337;31,19,7°, for latitudes by 9;12,43,48,15. The (cycles) of 25 (for) longitudes by 292;(33,57,21°, for latitudes by 6;38,)11,24,45 and the (number) of (intervals of) 248 for longitudes by 27;43,24,56°, for latitude by 2;43,28,34. Then add for longitudes 12;34,40,38°, for latitudes subtract 0;21,22,14,15; then count it off from Leo.

19. Lines 41 to 47 are a sloppy repetition of section 2 lines 15 to 29. Again the case of big steps is disregarded.

The last three lines concern the epoch. The value for the longitude is

$$\lambda_1 = \text{♌}\ 12;34,40,38$$

and is identical with the longitude we find for Nero 14 X 23 in L line [13]. This confirms our determination of the date of the new epoch.

For the "latitude" we find (line 49)

$$\beta_1 = \text{♌} - 0;21,22,14,15 = \text{♋}\ 29;38,37,45,45.$$

This gives us a possibility of checking once more the parameters for the latitude. From the example computed in No. 7 we know that $n = 99$ years and 176 days lie between Augustus —1 I 1 and Vespasian 1 VI 26. An additional 61 days lie between the first epoch and Augustus —1 I 1. The total number of days between the first epoch and Vespasian 1 VI 26 is thus found to be 36372 days, which are exactly 12 groups of 3031 days. The new epoch lies one ordinary step of 248 days before Vespasian 1 VI 26.

Thus we should obtain β_1 from β_0 by adding to $\beta_0 = ♋\ 17;47,20,40,45$ twelve times the value

$$k = 9;12,43,48,15$$

for one group and subtracting the value

$$b = 2;43,28,34,0$$

of one ordinary step. In this way one obtains for the new epoch 125;36,37,45,45 while the text gives

$$\beta_1 = ♋\ 29;38,37,45,45.$$

The discrepancy in the integer degrees is not surprising because we found already before that all values of β appear reduced modulo an unknown integer. The fractional part, however, we should expect to agree, but we find ;36,37,45,45 instead of ;38,37,45,45. I feel confident that this discrepancy must be explained by an error in the values given for β_0 and β_1. Either we must have

$\beta_0 = ♌ - 12;12,39,19,15 = ♋\ 17;47,20,40,45$
$\beta_1 = ♌ - 0;23(!),22,14,15 = ♋\ 29;36,37,45,45$

or

$\beta_0 = ♌ - 12;10(!),39,19,15 = ♋\ 17;49,20,40,45$
$\beta_1 = ♌ - 0;21,22,14,15 = ♋\ 29;38,37,45,45.$

The second pair, assuming miscopying $\iota\beta\ \iota$ as $\iota\beta\ \iota\beta$ seems to be more plausible than an error $\varkappa\lambda$ for $\varkappa\alpha$.

Knowing the date of the new epoch, Nero 14 = Nabonassar 815 X 23, we can again compute the elements of the mean moon by means of the Almagest. One finds

Longitude	♌ 3;47
Anomaly	340;51
Arg. of Lat.	349;42
Elongation	63;8

This time about 16 hours more would be needed to obtain the longitude λ_1 of R and this would also bring the anomaly closer to the apogee (349). Of real importance, however, is the value of the argument of latitude, which shows that this time the moon is very close to the "northern limit" of greatest positive latitude

(☊ 14;5). This time β_1 is almost ☊ 0°, thus close enough to the northern limit as not to exclude the possibility that this point is meant to be indicated.

Unfortunately this agreement seems to be accidental. If the motion of the northern limit was intended to be given by the values for the "latitude", we should expect for one ordinary step about 13;7°, for a group about 160;29°. These numbers show no relation to the values in the text: 2;43,... and 9;12,... respectively.

20. R, section 5 (lines 51 to 56). Translation.

On the nodes. {On the nodes.} Divide the total of years by 18, the remainder by 19, and the Egyptian months by 1;35, the days by 0;3,10. The total number.......... .....[.... the] cycles [of 1]8 make ....

21. According to the Almagest IV, 3 the mean movement of the moon in longitude is 13;10,34,58...° per day, whereas the argument of latitude increases by 13;13,45,39,...° per day. Consequently the nodal line recedes daily 0;3,10,41,...°. This value is abbreviated in R to 0;3,10. Consequently one obtains the following values for the retrograde movement of the nodal line

1 day....................	0;3,10°
1 month = 30 days........	1;35°
12 months = 360 days......	19°
18 · 12 months..............	— 18° (mod. 360)

If we want to know the movement of the nodal line during a time t of N years m months and d days, we first divide N by 18 and find a quotient α and a remainder β between 0 (included) and 18 (excluded). If each year had exactly 12 months the retrograde motion of the nodal line would be

$$-\alpha \cdot 18 + \beta \cdot 19 + m \cdot 1;35 + d \cdot 0;3,10.$$

Actually, however, we have disregarded $5\,N$ days of t. Thus the correct result would be

$$\nu = -\alpha \cdot 18 + \beta \cdot 19 + m \cdot 1;35 + (5\,N + d)\, 0;3,10.$$

All that is preserved in the text of section 5 is the rule that one should find β and form

$$\beta \cdot 19 + m \cdot 1;35 + d \cdot 0;3,10.$$

22. R, section 6 (lines 57 to 73). Translation.

Solstices and equinoxes which Ptolemy had observed in the year 463 [from the] death of Alexander. Summer solstice Mesore (XII) 11 to 12 7th hour of night. Add 92;30 days. This is the beginning of the observations. The autumn equinox Athyr (III) [9] at the first hour, approximately, after sunrise. Add 88;7,30 days. Winter solstice Mechir (VI) 7 4th hour of the day. Add 90;7,30 days. Vernal equinox Pachon (IX) 7 at the 1st hour, approximately, after noon. Add 94;30 days. It is the 3rd year of Aelius Antoninus. From the 4th year to the present year take (the total years) and divide by 4 and form the total of days; subtract for each year 0;0,12 and add the result to each of the observations.

Between lines 72 and 73 is written in small letters 35 and 30 of unknown meaning.

23. This section quotes four Ptolemaic observations of solstices and equinoxes for the year Antoninus 3 = Philip 463. Three of these observations are known from the Almagest. The fourth observation is probably supplemented by means of the known length of the seasons. Beginning with the summer solstice we have

	Almagest III,1	R
Heiberg 206,1	XII 11/12 2^h after midn.	XII 11/12 7^h of night
Heiberg 204,7	III 9 1^h after s. r.	III [9] 1^h after s. r.
Heiberg 205,1	IX 7 1^h after noon	IX 7 1^h after noon

The time intervals between these moments are supposed to be

92;30 88;7,30 90;7,30 94;30

according to Hipparchus and Ptolemy. The third number is erroneously given in line 65 as 95,30. The hours agree with these differences except for the first pair, whose difference is only 92;15. It also should be remarked that the first observation should actually be the fourth; in the present arrangement it would fall at the end of Antoninus 2 instead of Antoninus 3.

The above set of four dates is used for computing the dates of all subsequent solstices and equinoxes. Let N be the number of Egyptian years elapsed since Antoninus 3 = Augustus 169. Because four Egyptian years are one day shorter than four Julian years, we divide N by 4 and obtain a quotient α and a remainder β less than 4. Every group of four years adds one day to the date of the Egyptian calendar. Because Antoninus 3 is a leap year in the Alexandrian calendar, the increase in the date should be $\alpha + 1$. The text begins the counting of years with Antoninus 4, obviously in order to add only α days, but fails to remark that the initial dates must be raised by 1 in order to correspond to the year Antoninus 4.

Because according to Almagest III,1 the tropical year is $0;0,12^{d}$ shorter than the Julian year (Heiberg p. 208), we have to subtract $N \cdot 0;0,12$ (lines 71 to 73) in order to obtain the moment of the solstices or equinoxes.

24. R, section 7 (lines 75 to 84). Translation.

Remaining years of Aelius Antoninus 1[9]

221	Commodus	32	[year] 1	190	294
246	Severus	25	year 1	220	
250	Anosius	4	year 1	247	
263	Alexander	13	year 1	251	
266	Maximinus	3	year 1	264	
272	Gordianus	6	year 1	267	
278	Philippus	6	year 1	273	
2[80]	Decius	2	year 1	279	
[282]	Gallus	2	year 1	28[1]	

. . . .

Reading. In line 75 also κ seems possible instead of ι̣[θ].

25. The restoration 1[9] in line 75 is based on the assumption that the "remaining years" of Antoninus are counted from year 4, mentioned in the preceding section (line 69) as the year from which to count for the determination of the solstices and equinoxes. Because Antoninus ruled 23 years, 19 would be the remaining number. If one reads 20, the remaining years would be counted from year 3. The editors of R restored 1[6] for reasons unknown to me.

The years in the subsequent regnal canon are to be interpreted as follows

$$\begin{aligned} \text{Augustus } 221 &= \text{Commodus } 32 \\ \text{Commodus } 1 &= \text{Augustus } 190 \\ &= \text{Philip } 190 + 294. \end{aligned}$$

Similarly for all following lines.

This concordance between the Era Augustus and the Era Commodus is paralleled in the change of era from section 1 to section 3. The Era Philip might have been added in order to coordinate dates with the "Handy Tables" of Ptolemy[1].

26. We may now summarize the contents of R as follows. The main purpose is to give rules for the computation of dates and longitudes of the moon such that the apogee falls at the same time of the day (e.g., noon). At the base of the method which is followed there lie Babylonian periods for the restitution of the lunar anomaly. Similar rules are given for "latitudes" though the significance of this term is still unknown. Simple rules for the movement of the nodes and for the dates of solstices and equinoxes are added. Here Ptolemaic observations are quoted and his value for the precession is used. This is obviously the reason for the note "treatise of Ptolemy" written on the other side of R. The main part of R, however, follows methods totally different from those of the Almagest.

We see now that R is the central text which connects Demotic and Greek treatises which span at least the 400 years from the early second century B.C. to the middle of the third century A.D. The existence of the methods used by these papyri would never have been conjectured either from the Almagest or from the Seleucid Babylonian texts. It is an entirely new phase of "Hellenistic astronomy" of which we see here a first, however fragmentary, chapter.

[1] Ptolemaeus, Opera astron. min., ed. Heiberg p. 160,22 f.

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